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13. ABSTRACT *(Maximum 200 words)*

Models of turbulence for complex engineering flows need to be able to span the full range of deformation rates from slow, where eddy viscosity models (*e.g.* k-ε) relating the turbulent stress to the mean strain *rate* are valid, to rapid deformations where the stresses are determined by the *amount* of strain. In order to build a one-point turbulence model that can match Rapid Distortion Theory (RDT) when appropriate, new concepts are needed to parameterize the turbulence structure. This research, which builds upon these new concepts, develops a new structure-based model for RDT of homogeneous turbulence that will be used as the backbone of a more general model for both slow and rapid distortion in general turbulent flows. The model has been tested against exact RDT for a very wide variety flows involving various complex combinations of mean strain and rotation. Adequate agreement is found in all cases and excellent agreement in most. The work is a key step towards a more general, robust engineering model for predicting turbulent flows.

14. SUBJECT TERMS
Turbulence modeling, rapid distortion, homogeneous turbulence, one-point modeling, turbulence structure

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Final Technical Report
Structure-Based Turbulence Model
AFOSR-91-0216

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1. Background

In order to construct a robust engineering model for complex turbulent flows, it is necessary to have a good one-point model that can correctly predict the response of turbulence to rapid mean deformation. It is well established that Rapid Distortion Theory (RDT) is the correct limiting theory for this case. RDT is a linear theory in which the non-linear interactions of the turbulence with itself are neglected in comparison to the response of the turbulence to the rapid mean deformation. But even though the equations for the velocity fluctuations are linear, a closed set of equations for the Reynolds stresses can not be derived from the RDT equations because the pressure fluctuations are non-local. Hence, in a one-point theory, modeling is needed even for RDT.

Concepts of *dimensionality* and *componentality* are fundamental to the present work. If the turbulence is independent of the x_1 direction, then the turbulence field is a 2-dimensional (2D) field. The streaky near-wall region of a boundary layer is very nearly 2D. However, 2D turbulence could have one, two, or three non-zero velocity components, and hence could be 1C, 2C, or 3C. The RDT limit for shear flow is a 2D-1C field, and the wall region of a boundary layer is very nearly such a flow. The property of *material indifference* to frame (or mean) rotation is a feature of 2-*dimensional* turbulence with its axis of independence aligned with the axis of frame (or mean) rotation. A good turbulence model should exhibit this property, especially if it is to be used in turbomachinery.

The Reynolds stresses $R_{ij} = \overline{u_i' u_j'}$ provide information on the *componentality* of the large-scale turbulence. Hence, a model that carried R_{ij} could not properly identify the *dimensionality* of the large-scale turbulence, and hence can not properly exhibit material indifference. In order to construct a turbulence model that can handle rotation properly, it is necessary to get *dimensionality* information into the model. This means that the model must contain information about the *structure* of the large-scale turbulence. The development of a way to do this, and the application of these ideas in constructing a new turbulence model, forms the focus for this work.

2. Objectives

The objectives of this work were as follows:

- To define and explore new tensor concepts for characterizing the structure of the energy-containing eddies in turbulent flows.

- To develop the theory for the rapid evolution of these tensors, and to use this information to develop new physical understanding of the effects of mean deformation on turbulent stresses and structure.

- To develop a one-point model for predicting the Reynolds stress history of homogeneous turbulence under arbitrary rapid mean deformations, for use as the backbone of a more general turbulence model that can handle both slow and rapid deformations in complex flows of technical interest.

2. Accomplishments

A new set of tensors characterizing the turbulence structure were defined in terms of the vector stream function of the turbulence, which is defined by

$$u_i = \epsilon_{ijk}\psi'_{k,j} \qquad \psi'_{i,i} = 0.$$

Three new tensors were defined as follows:

- Dimensionality: $D_{ij} = \overline{\psi'_{n,i}\,\psi'_{n,j}}$
  If the turbulence is 2D and independent of x_1, then $D_{11} = 0$.

- Circulicity: $F_{ij} = \overline{\psi'_{i,n}\,\psi'_{j,n}}$
  If the turbulence consists of vortices with axes in the x_1 direction, then F_{11} is the only non-zero element.

- Inhomogeneity: $C_{ij} = \overline{\psi'_{i,n}\,\psi'_{n,j}}$
  If the turbulence is homogeneous, then $C_{ij} = 0$.

Values for these tensors were determined using the data base of direct numerical simulations available at the NASA/Stanford Center for Turbulence Research. These verify that the tensors do indeed reflect the turbulence structure.

Important algebraic relations relating these tensors were derived. For homogeneous turbulence, one finds that

$$D_{ij} + R_{ij} + F_{ij} = q^2\delta_{ij}$$

which shows that only two of R_{ij}, D_{ij}, and F_{ij} are linearly independent, and that it takes two of these tensors to fix the turbulence state. Hence a turbulence model should incorporate information from two of the tensors.

Dynamical equations for the evolution of these tensors under RDT for homogeneous turbulence were developed. The use of these equations as the basis for a turbulence model was explored, and considerable physical insight was obtained, particularly in regard to the effects of mean rotation.

The correct general form of the spectrum tensor for axisymmetric turbulence was derived, and RDT was used to show that a critically important term was omitted by earlier classical developments. This term is a result of the breaking of reflectional symmetry by mean rotation. As a result, another fully symmetric third-rank tensor Q^*_{ijk}, the *stropholysis*

(Greek for breaking by rotation) would be needed in addition to the two second-rank tensors in order to handle rotation effects correctly.

With the recognition that the complexity of a turbulence model carrying two second-rank tensors and a third rank tensor would probably render it unappealing for engineering analysis, ways to capture the essential physical information in a simpler way were explored, and a remarkably effective way was found. This led to a simple ***structure-based turbulence model*** (for RDT) involving only one tensor, an " eddy axis tensor" a_{ij}, and two scalar parameters. One parameter ϕ relates to the relative energy associated with motions round the eddy axis (vortical turbulence) and motion along the eddy axis (jetal turbulence). The other parameter γ relates to the correlation of the vortical and jetal motions, and carries the information from the stropholysis tensor. The model involves an algebraic equation giving the stresses in terms of the eddy axis tensor, the direction of the mean rotation vector, the two scalar parameters, and the turbulence energy, and transport equations for the scalar parameters, which were derived from the Navier Stokes equations and required very minimal and very robust modeling. The dimensionality and circulicity tensors and the stropholysis tensor are also given by algebraic relationships involving a_{ij}, ϕ, and γ.

The model was tested against exact RDT for approximately 50 different combinations of mean strain and mean rotation, with performance as follows:

- Essentially exact results are obtained for arbitrary irrotational mean deformations.
- The effects of mean rotation, including material indifference and phase randomization by rapid rotation, are captured correctly. No other turbulence model can handle these flows properly (because no other model contains dimensionality information).
- The alteration in the stresses cause by mean rotation (swirl) of strained turbulence are very strong, and are captured correctly by the model.
- The effects of rapid compression or expansion of anisotropic flows are captured correctly. This will be particularly important in dealing with shock-boundary layer interactions.

In order to carry out exact RDT calculations, a new Particle Representation Model (PRM) was developed. In this model, one calculates the evolution of a few representative particle clusters, and from these calculations determines the statistical tensors of interest described above.

The results of this project are described in complete detail in the companion report by Kassinos and Reynolds (1994), filed separately with the AFOSR and available from the DOD or Stanford University. Here we present only a brief selection of the results to show the capabilities of the model.

Fig. 1 shows the Reynolds stress anisotropy $b_{ij} = (R_{ij} - q^2\delta_{ij})/q^2$, where $q^2 = R_{kk}$, as a function of the amount of strain for turbulence that is subjected to a combination of mean rotation and incompressible axisymmetric contraction (similar to a swirling nozzle flow). The mean swirl rate is initially five times the axial strain rate and increases as a result of the strain, and over time the turbulence becomes very nearly 2D and independent of

the axis of rotation. Note that the model is in excellent agreement with the exact RDT, and that the stresses are quite different than what would be predicted by considering the strain rate without rotation (as in eddy viscosity models).

Fig. 2 shows the stress anisotropy for one-dimensional axial compression of a swirling flow (similar to a vortex passing through a shock). Note that the model correctly captures the isotropization produced by the compression, whereas an eddy viscosity model in which the stresses are determined by the strain rate will not.

Fig. 3 shows the model predictions for shear in a rotating frame (similar to flows on turbomachinery blading), where the frame rotation rate is equal to but opposite to the shear rate. Note that the streamwise stress $r_{11} = R_{11}/q^2$ is very small in this case and in the limit vanishes, whereas for shear in a non-rotating frame the r_{11} component is very large and in the limit of infinite deformation goes to unity. Eddy viscosity models do not predict the correct effects.

Fig. 4 shows the normalized shear stress r_{12} after a particular amount of rapid shearing for shear flow in a rotating frame, as a function of the ratio of frame rotation rate to shear rate. Note the excellent agreement with RDT. The RDT and model also agree quite well with the Large Eddy Simulation by Bardina, indicating that the conditions of his simulation were appropriate for rapid approximations.

3. Personnel

The research formed the basis for the PhD dissertation of S. C. Kassinos, and was conducted under the guidance of Professor W. C. Reynolds.

4. Continuation and Transitioning.

Work on the extension of the model to cover moderate and slow deformations is continuing under new AFOSR support. Dr. Kassinos and two PhD students are involved in this work. One of the students, a former Pratt and Whitney employee, will put the new model into applications codes of the type used in the turbomachinery industry, and his contacts with this industry should assist in its rapid evaluation and deployment.

Professor Steven Pope at Cornell has used our Particle Representation Model of RDT to improve his Monte-Carlo pdf modeling (used in combustor analysis in conjunction with many companies), making it consistent with RDT when RDT applies.

Various elements of the ideas were presented over the past several years in meetings of the American Physical Society's Fluid Dynamics Division. The first presentation of the new model was given as keynote lecture at a Symposium held in 1994 on the centenary of Osborne Reynolds' paper in which his stresses were first introduced and a condensed version of this method will appear in a commemorative volume of the Proceedings of the Royal Society. A series of articles on this work are now being prepared for submission to the Journal of Fluid Mechanics.

5. Publications

Kassinos, S. C. & Reynolds, W. C. 1994 A structure based model for the rapid distortion of homogeneous turbulence. *Report TF-61*, Thermosciences Division, Department of Mechanical Engineering, Stanford University, December.

Reynolds, W. C. & Kassinos, S. C. 1994 A one-point model for the evolution of Reynolds stress and structure tensors in rapidly deformed homogeneous turbulence. Osborne Reynolds Centenary Symposium, UMIST, Manchester, to appear in *Proc. Roy. Soc. A*.

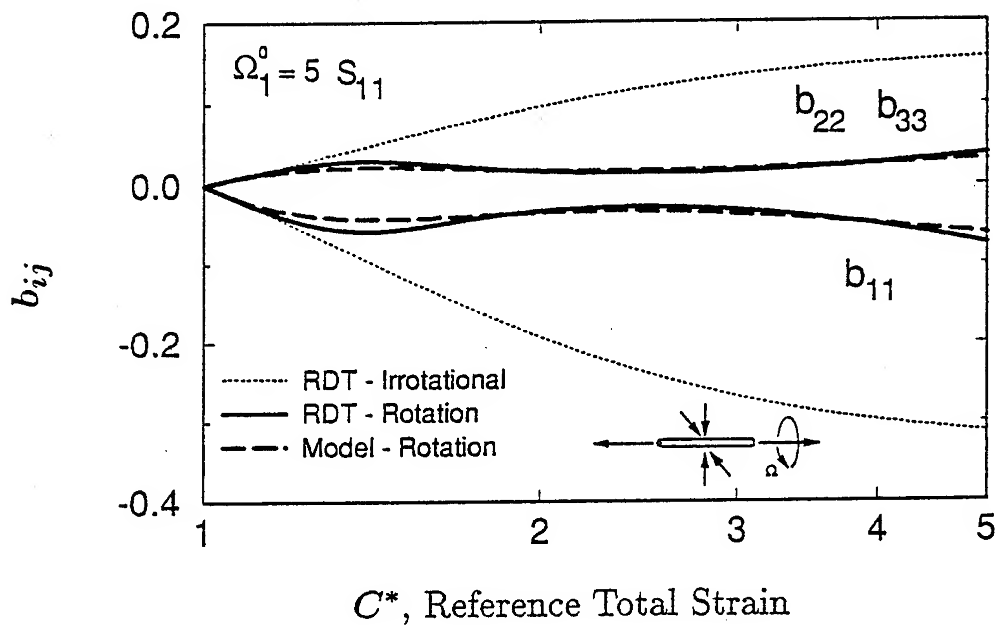

Figure 1. Stress anisotropy for axisymmetric contraction with mean rotation.

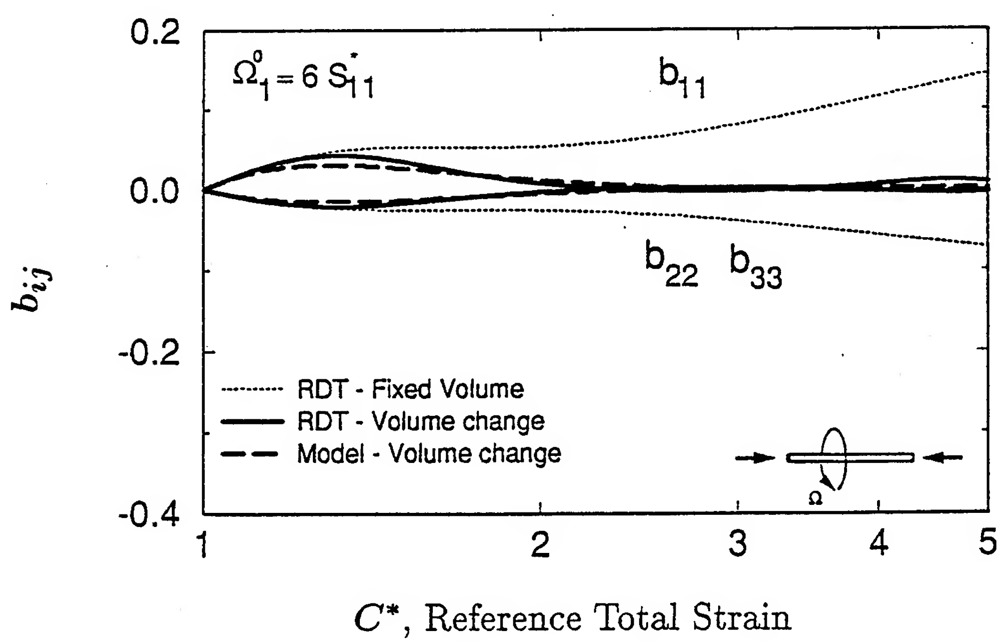

Figure 2. Stress anisotropy for axial compression with mean rotation.

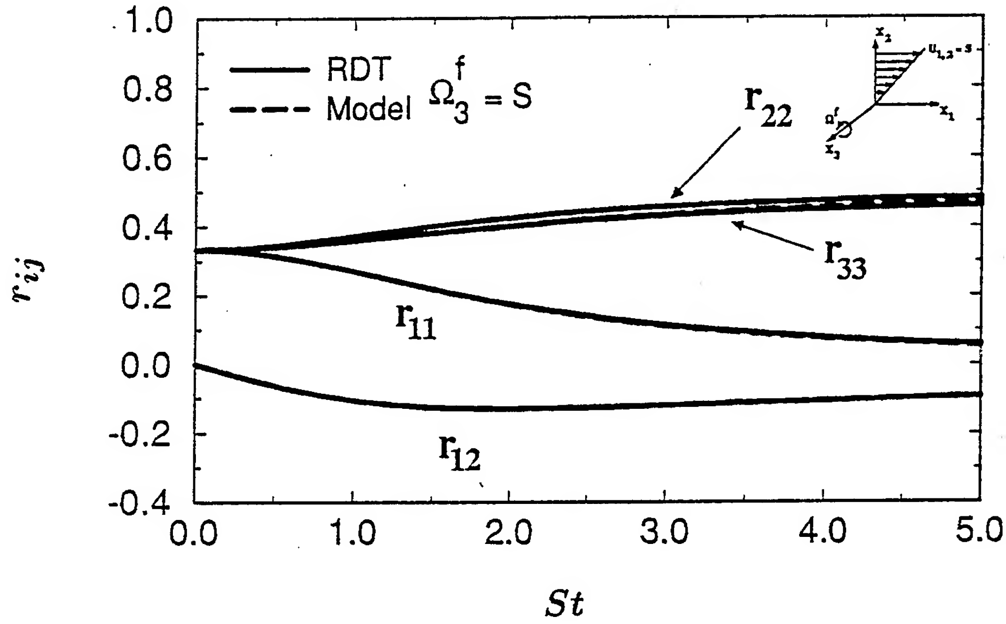

Figure 3. Non-dimensional stresses for shear in a rotating frame.

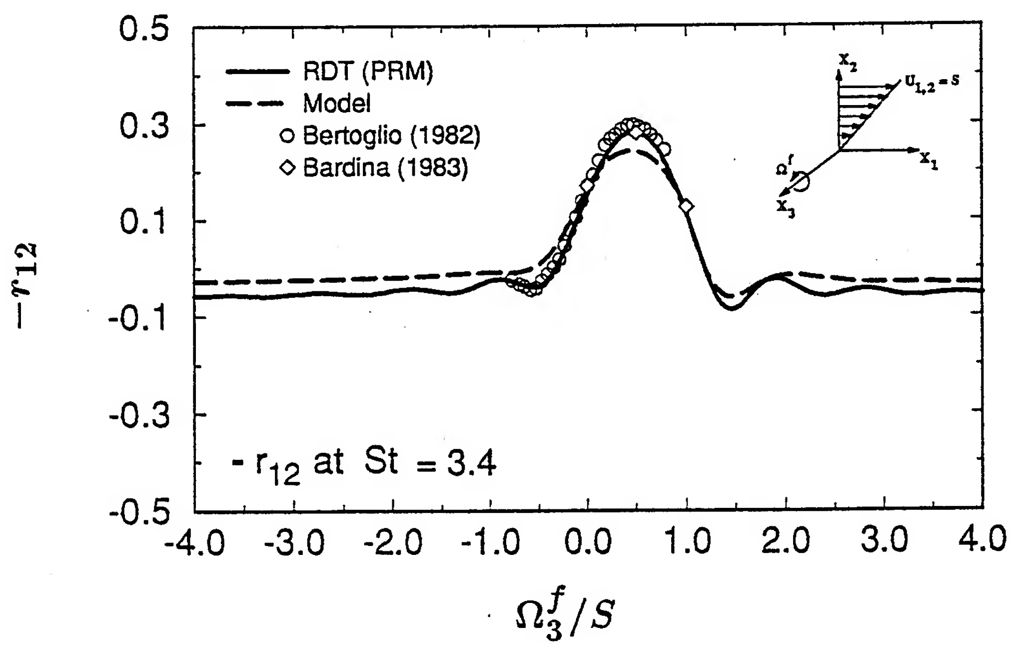

Figure 4. Shear stress for shear in a rotating frame.